

# Choosing maintenance analysis techniques

## Understanding the differences between Cost Minimisation Algorithms and the RCM concepts developed by Nowlan and Heap (1978)



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<b>BACKGROUND .....</b>	<b>3</b>
<b>AIM.....</b>	<b>3</b>
<b>DEFINITION OF A COST MINIMISATION ALGORITHM PROGRAM.....</b>	<b>3</b>
<b>USING TRUE RCM TO DEFINE CONDITION MONITORING INTERVALS .....</b>	<b>4</b>
FAILURE PATTERNS.....	4
OUTLINE OF THE TRUE RCM APPROACH TO CONDITION MONITORING.....	5
<b>COST MINIMISATION ALGORITHMS .....</b>	<b>6</b>
ASSESSING THE CMA ASSUMPTIONS.....	6
<i>Maintenance is a variable</i> .....	6
<i>Summary</i> .....	8
DATA INTEGRITY.....	8
<i>Statistical Approaches</i> .....	8
<i>Summary of data inputs</i> .....	8
PRACTICALITIES OF IMPLEMENTING CMA ANALYSES.....	10
THE WIDER IMPLICATIONS OF UNCERTAIN MAINTENANCE POLICIES.....	10
<i>Reliability Engineering Focus</i> .....	10
<i>Planning and Scheduling</i> .....	11
<i>Spares Assessing</i> .....	11
<b>CONCLUSION .....</b>	<b>12</b>
<b>REFERENCES:.....</b>	<b>13</b>
<b>EXPLANATION OF DETERMINING THE INTERVAL OF INSPECTION.....</b>	<b>14</b>
<b>MATHEMATICS TO DETERMINE THE RELATIONSHIP BETWEEN PLANNED MAINTENANCE COSTS AND UNPLANNED MAINTENANCE COSTS.....</b>	<b>15</b>
<i>Hypothesis</i> :.....	15
<i>Definitions</i> :.....	15
<i>Variables</i> .....	15
<i>Assumptions</i> :.....	16
<i>Analysis</i> .....	16

## **BACKGROUND**

Maintenance analysis has developed significantly over the past 20 years due primarily to the following:

- The abolition of traditional views of maintenance following the works of Nowlan and Heap (1974 - 1978) in the area of Reliability Centred Maintenance (RCM),
- Access to computerisation as a tool to administer and analyse maintenance requirements, and
- The ever increasing need for capital intensive industries to become more cost competitive and reduce the risk of a major industrial accident.

Over the years, there has been considerable growth in the number of maintenance analysis techniques or methods available. There has also been a recent rise of concern in the maintenance industry about the quality and legitimacy of many of these.

In August 1999 the SAE published a standard titled “Evaluation Criteria for Reliability – Centered Maintenance “ (SAE JA1011, 1999).

This purpose of the standard is stated in its foreword. It reads as follows:

*“... the widespread use of the term “RCM has led to the emergence of a number of processes that differ significantly from the original, but that their proponents also call “RCM”. Many of these processes fail to achieve the goals of Nowlan and Heap, and some are actively counterproductive.”*

*“This document is intended for anyone who wishes to ascertain whether any process that purports to be RCM is in fact RCM. It is especially useful to people who wish to purchase RCM services (training, analysis, facilitation, consulting or and combination thereof).”*

There is no compulsion for any organisation to use an SAE compliant maintenance analysis process. However, because there are some maintenance analysis techniques that are counterproductive, buyers should, at least, understand exactly what they are purchasing and the potential consequences of implementing decisions based on what they have purchased.

## **AIM**

The aim of this paper is to present a review of Cost Minimisation Algorithms (CMA's) in the context of comparing them with the original works of Nowlan and Heap (1978) and the subsequent SAE Standard JA 1011 dated August 1999.

The purpose of this paper is to provide information to purchasers of RCM analysis tools so that they may make informed decisions. Therefore, the paper does not stop at the comparison; it discusses the practical and business implications of the variations.

## **DEFINITION OF A COST MINIMISATION ALGORITHM PROGRAM**

For the purpose of this paper a Cost Minimisation Algorithm (CMA) program is a program that is characterised by two significant points.

1. CMA's consider that more frequent Preventive Maintenance (PM) will be more expensive in maintenance costs but more rewarding in increased reliability. PM is therefore a variable that has some optimal frequency. Because of this, using a CMA it is possible that the recommended interval for condition monitoring, for example, is at a time greater than the PF interval<sup>1</sup>. This is not possible using any form of RCM logic.
2. The intervals for all types of maintenance are best found using statistical modeling methods.

Whilst many CMA's will originate with the same failure mode analysis as RCM, they will diverge when the maintenance task decision logic is applied.

## USING TRUE RCM TO DEFINE CONDITION MONITORING INTERVALS

### Failure Patterns

Until the mid 1970's, the accepted theory of maintenance was that all equipment had a safe or economic life and that this life could be defined. The role of maintenance was, therefore, to remove assets from service before failure. For this reason aircraft, for example, were maintained largely by overhaul with the majority of components being scheduled for removal at a defined time regardless of their condition.

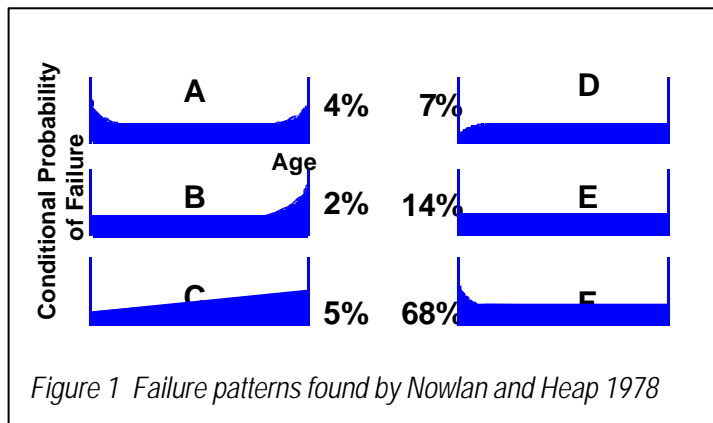


Figure 1 Failure patterns found by Nowlan and Heap 1978

The data collected over four years from the aircraft industry<sup>2</sup> by Nowlan and

Heap disproved the theory that all equipment had a safe or economic life. Nowlan and Heap's analysis showed that 68% of aircraft failures occurred as infant mortality with a further 14% being random. It was clear from this information that doing more overhaul or intrusive maintenance had the potential to increase plant failure rates rather than reduce them. Figure 1 Refers.

This fundamental finding led to the formulation of a maintenance analysis method named, by Nowlan and Heap, as Reliability Centered Maintenance (RCM)<sup>3</sup>. The conclusions of the Nowlan and Heap work have been proven, in practice, beyond doubt (Moubray 1997).

<sup>1</sup> The interval between the point in time where a potential failure condition can be first discovered and the point in time where it has functionally failed.

<sup>2</sup> One of the few industries where data collection is reliable.

<sup>3</sup> Hereafter called true RCM.

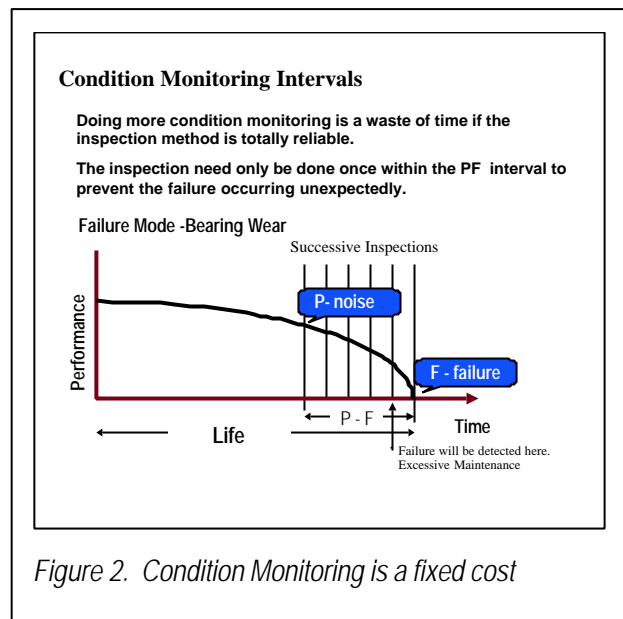
## Outline of the True RCM Approach to Condition Monitoring

RCM is a zero-based analysis that starts by identifying all the equipment functions followed by listing functional failures. The process continues by listing all likely failure modes the plant may experience and analysing each in turn to decide what, if any maintenance is feasible and cost effective. This part of the analysis is often referred to as “failure mode analysis”.

In reaching decisions regarding task intervals and effectiveness, true RCM relies more on empirical methods than complex statistical analysis. This can be explained by first understanding the conclusion drawn by Nowlan and Heap, that the primary defence against failures is condition monitoring and that condition monitoring relies on rates of decay of equipment condition. Figure 2 defines the PF interval as the interval between the point when an indication of a potential failure can be first detected (P) and the point where the equipment is said to have functionally failed (F). Figure 2 also demonstrates, that if one inspection is effective in detecting a given potential failure mode and the inspection is conducted at a frequency less than the PF interval, then any such failure mode will be detected before it occurs. More frequent inspections will not improve the chances of detection whatsoever<sup>4</sup>. Conversely, inspections performed at intervals greater than the PF interval will result in some unexpected failures.

Without spending a great deal of time on condition monitoring intervals, the following points should be understood in the context of this discussion:

- In most cases in industrial plant, the tradesmen or the operators best understand the PF interval because it is an experiential and somewhat abstract construct assessed by these people on a daily basis.
- The PF curve rarely exists in databanks<sup>5</sup>.
- Contrary to many beliefs, the interval of condition monitoring is not influenced by failure pattern nor the probability of failure. This is explained neatly by an example at Appendix A (Netherton, 2001).



<sup>4</sup> It is important to note that condition monitoring is only appropriate for failure modes that provide some identifiable warning during their path of deterioration.

<sup>5</sup> It relies on knowing when the symptom first became evident and then running the equipment to near destruction.

### Assessing the CMA Assumptions

#### Maintenance is a variable

Most CMA's were developed independently of the work of Nowlan and Heap and without the benefit of the failure patterns (Figure 1) found in the aviation industry. They were developed on the premise that more maintenance would be more expensive in maintenance costs, but more rewarding in reliability. Based on this premise, it was logical to postulate that there was some minima associated with this relationship.

CMA's therefore rely on a costing algorithm to statistically model the interval of preventive maintenance that would provide the overall lowest business cost. Figure 3 shows a typical view of the relationship between costs and inspection interval as suggested by a CMA process.

The difficulties with such a premise are discussed below in more detail.

#### Scheduled Replacement (Overhaul)

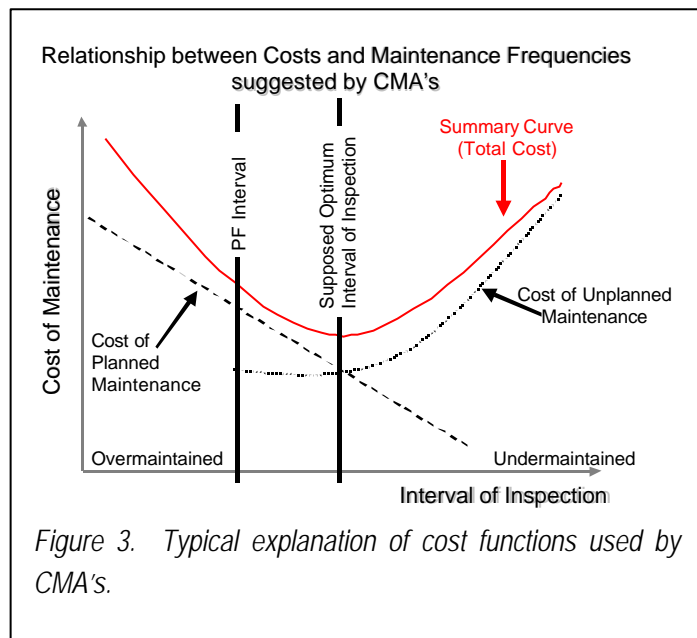
Nowlan and Heap (1978) demonstrated that in the aircraft industry, 68% of failures occurred shortly after maintenance (Figure 1). This was likely to have been caused by damage sustained during the intrusion, the fitting of defective equipment, or the incorrect fitting of serviceable equipment.

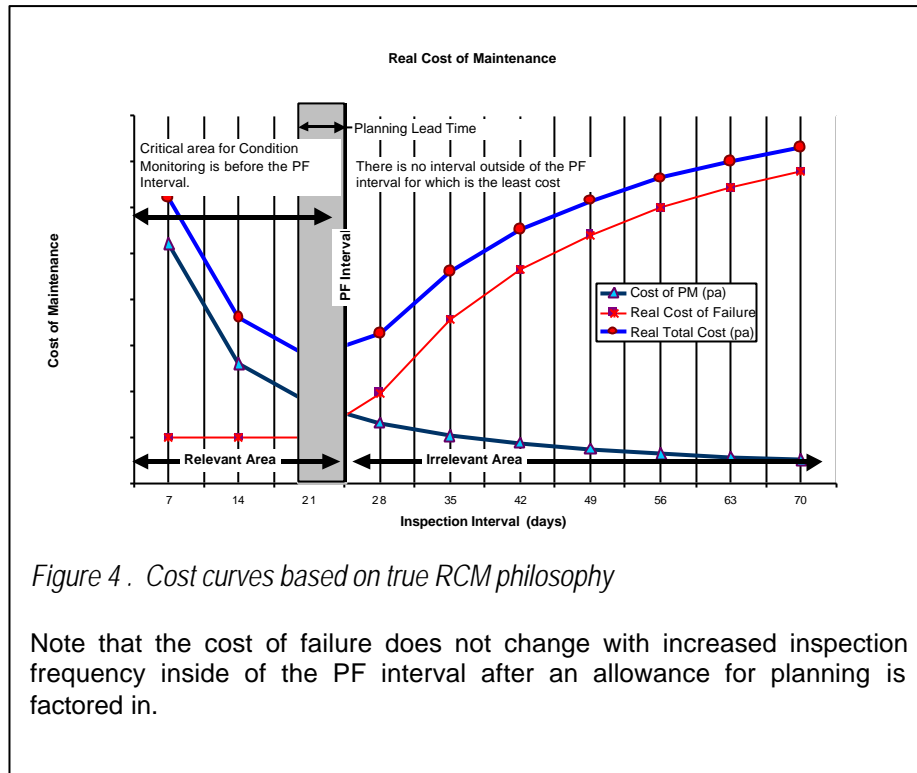
It follows that in such cases, doing more maintenance will increase the rate of failures rather than reduce it. This directly conflicts with the CMA premise and their subsequent algorithms.

#### Condition Monitoring

Condition monitoring is feasible if a failure mode gives some warning that the component condition is deteriorating. To be successful at condition monitoring, one needs to inspect at a frequency shorter than the fastest rate of deterioration. If the time between the point of first indication (P) and the time of the failure (F) is known as the PF interval, then one needs to inspect for the deterioration at an interval less than the PF interval. (Figure 2 refers)

The first point is that; if inspections are reliable and completed at an interval less than the PF interval, then there is no benefit from increasing inspection frequencies. More inspections will only add value if the method is not 100% reliable which in statistical terms is known as statistical confidence.





In assessing the interval of inspection for inspection methods that are not 100% reliable, it can be shown that performing more than one inspection inside the PF interval will increase the chances of identifying a potential failure<sup>6</sup>. This logic means that it would be absurd to think that greater confidence will come from fewer inspections.

The second point is that it can be shown mathematically at Appendix B, that for inspection intervals greater than the PF interval the least cost interval will be either infinite or converge to the PF interval itself. This depends on whether the cost of PM plus the planned rectification is greater or less than the cost of failure over a significant period. If the cost of PM plus the planned rectification is greater than the cost of failure, then there is no interval of inspection that will change that. If the cost of PM plus the planned rectification is less than the cost of breakdown, then the problem is simplified. In this case, the interval must be a point somewhere inside the PF interval. In practice, there needs to be some allowance for planning time. A reduced interval of inspection is generally chosen to provide for this. Figure 4 illustrates this point.

This means two things:

1. There will never be a least cost interval outside the PF interval other than infinity. An inspection interval of infinity equates to a maintenance policy of "no scheduled maintenance".
2. For any inspection task to be cost-effective, it will always need completion within the PF interval. The primary determinant of the inspection interval will be the

<sup>6</sup> If rolling a dice and counting a number five or less as a success, the more the dice is rolled, the less will be the chances of failure.

required lead-time to plan the inspection. The failure pattern and MTBF are second order effects that will not change these facts.

## **Summary**

This assessment of CMA philosophies suggests that they have serious conflicts compared to the findings of Nowlan and Heap. For this reason, such CMA's do not conform to SAE JA1011 on many counts, with the most serious variations found in Sections 5.6 and 5.7, which deal with failure management policies and describe how true RCM defines intervals of inspection.

## ***Data Integrity***

### **Statistical Approaches**

As CMAs are a statistical packages which, infers a reliance on valid data and accurate input variables.

In industry, such failure data rarely exist. By virtue of the fact that the objective of maintenance is to prevent failure, those data will never exist in a marginally successful business (Resnikoff, 1978). Any analysis based on failure statistics will be based on a significant number of guesses that compound into a result that has almost no level of statistical confidence, and is of questionable value.

### **Summary of data inputs**

Most CMA cost curves require the following inputs:

- The cost of actual failure, or breakdown cost;
- The cost of the preventive maintenance primary action - this is the routine inspection, service or condition or performance measurement activity;
- The cost of the preventive maintenance secondary action - this is the preventive or corrective repair that is done as a result of an observation or warning condition noticed as a result of conducting the primary action; and;
- The probability of failure for the equipment being maintained based on its life characteristic at the frequency of the primary action, which is designed to detect and prevent that failure.
- The probability that the condition based task will detect the symptom given that it exists.

### **Cost inputs are highly subjective**

In the practical environment, all costs are subject to opinion regarding the following assumptions:

- Whether to include overhead in labour costs or at the other end of the scale to consider that some labour is free as it is already employed in the area.
- Whether lost time can be recovered by overtime or whether sales are actually lost by breakdowns.
- Whether the impact of downtime is constant through the year or a significant variable.

- The consistency of the impact on the time to repair which is known to vary according to the severity of damage and the availability of spares and labour.

The data input for the costing information is, in practice, always highly subjective and is therefore likely to produce results of low credibility.

The other problem arises when attempting to cost safety, environmental, or commercial consequences, as these factors should be assessed as risks not costs.

#### MTBF will always be a guess

MTBF at the failure mode level will be a guess in almost all applications. This is primarily due to the following:

- The inability of most organisations to record accurate data; first of all at the component level and more importantly at the failure mode level,
- The objective of maintenance is to prevent failures before they happen thus eliminating the data source regardless of the best intentions of the collection methods,
- The likelihood that some components will be discarded with parent equipment when another part of the parent equipment fails. An example is a pump bearing which may be discarded when the pump is replaced due to a worn casing thus leading to inaccurate bearing MTBF calculations.

#### MTBF is not always a constant

Even though most CMAs require failure pattern inputs against failure modes, most then manage that data in such a way that they consider the MTBF to be constant through life. This is profoundly incorrect and in reality only true for random failures which account for a small proportion<sup>7</sup> of failures. To use the CMA algorithm correctly would require the constant updating of the MTBF as the asset aged and the probability of failure changed. This in turn would imply increasing the frequency of maintenance inspection over the equipment life; a situation that most CMA's do not prescribe. The oversimplification takes the logic into a statistical realm where the analysis becomes averaged to the extent that it makes no sense and where a common sense approach, based on fewer and more concrete factors, are far more successful.

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<sup>7</sup> Random failures were found to be 14% of the total failure pool of aviation failure history studies by Nowlan and Heap (1978)

## **Practicalities of Implementing CMA Analyses**

Maintenance in industrial plant is often a craft and an experiential construct. The shop floor personnel who are engaged to perform and analyse the maintenance requirements are not usually interested nor skilled in statistical methods. Setting the foundation of the asset management philosophy on a statistical framework usually results in a disinterest from the shop floor and poor buy-in.

Consequently, any shop floor suggestions to change the program are likely to be withheld. This is because the shop floor people who wish to make them, have no confidence in their logic and are not equipped to present a valid statistical case. In short, statistical methods preclude the involvement of the people who understand the equipment the best and who are in a position most able to make a difference to maintenance improvement. These are the people who operate and maintain the equipment.

The result is that analysis using most CMA's becomes a backroom activity often performed by contractors or engineers. Whilst equipped with the best intentions, these people are usually unaware of most of the specific practical issues faced by the maintenance team. As changes to the existing maintenance program are likely to be based on illogical algorithms, sweeping assumptions and poor data, the revised policy can make no sense to the shop floor people. They then tend to rebel against the changes rather than embrace them.

The use of statistical maintenance analysis methods creates a confused and frustrated maintenance organisation with an attitude of disempowerment rather than being involved in a framework of reliability improvement based around common sense and practical approaches. This fact alone can, and has, driven a maintenance organisation backwards quickly.

## **The Wider Implications of Uncertain Maintenance Policies**

If, for some unknown reason, the interval of condition monitoring were justified at an interval greater than the PF interval, then this would mean that the maintenance department was setting out to experience failures. This vague approach has several negative effects on other vital maintenance processes. These are discussed below:

### **Reliability Engineering Focus**

DuPont (Ledet, 1994) has found that the major gains in plant uptime come from reliability engineering rather than improving maintenance. Figure 5 shows the relationship between these factors. Maintenance departments should therefore aim to reach a point in time where their sole focus on reliability is in the area of defect elimination. To do this

Strategy	Change %	Uptime %
<b>Reactive</b>		<b>83.5%</b>
<b>Planning Only</b>	<b>+ 0.5%</b>	
<b>Scheduling Only</b>	<b>+0.8%</b>	
<b>Preventive / Predictive Only</b>	<b>-2.4%</b>	
<b>All Three Strategies</b>	<b>+5.1%</b>	<b>88.6%</b>
<b>Plus Defect Elimination</b>	<b>+14.8%</b>	<b>98.3%</b>

*Figure 5 – model produced by Ledet (1994) for Dupont and used in the Manufacturing Game [www.manufacturinggame.com](http://www.manufacturinggame.com)*

requires a disciplined and valid maintenance program and processes where every unplanned failure is assessed against the maintenance strategy / policy for that failure mode. Using true RCM, this is simple; either the failure will be expected to be removed before it occurs or it will be allowed to occur on purpose.

With most CMA's, there will never be any confidence that a plant failure was an unusual event or an expected outcome. This leaves any remedial activity or reliability engineering on shaky ground and suggests that the chances of being successful will be lower than what could be if true RCM were used as the maintenance logic.

## **Planning and Scheduling**

No amount of quality planning and scheduling can compensate for a badly focussed maintenance program. This fact arises for two simple reasons:

1. If the prescribed maintenance is overly intrusive and / or counterproductive, the absurd situation occurs where the better planning and scheduling becomes; the more the business will suffer. This is to say that the planning department will become better at doing things that are counterproductive.
2. The more that economically preventable failures result in unexpected failures<sup>8</sup>, the more that planning resources will be tied up in crisis management. This situation unnecessarily consumes their time and the time of other scarce labour resources.

## **Spares Assessing**

The major considerations in defining a maintenance spares policy are the lead-time to obtain the spare and the ability of the maintenance department to predict the failure<sup>9</sup>.

Organisations that have their maintenance under control and are experiencing a high ratio of planned to unplanned work can, without taking any greater risks, reduce their spares holding far more than those that are in a reactive environment. This is because in a reactive environment, the ability of the maintenance department to predict the failure is close to zero hence spares must be carried for all failures regardless of the predictability of failure.

Stores management in these situations behaves like supermarkets that need to carry vast quantities of product lines and then hope that there is not an unexpected rush on any particular item. Organisations that have effective condition based programs that are based on two options;

1. "Prevent" or
2. "Run to failure"

know that their investment in spares should be directed to those failure modes that are likely, random and can not be predicted by monitoring condition. Investment would be minimal in spares whose needs will be predicted in time for the parts to be

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<sup>8</sup> Due to insufficient condition monitoring or infant mortality caused by unnecessary overhaul for example.

<sup>9</sup> There are many other factors but these are become second order effects in an environment where the maintenance program is highly effective.

purchased and the job planned. It will be instead directed more to spares with failures that exhibit instantaneous demise with a random distribution.

As CMA algorithms set out to allow failures, this ensures that an organisation using such CMA's will not minimise its spares holding and suffer from either:

- Unnecessarily high rates of stock out or
- High levels of inventory.

Any statistical assessment undertaken on these terms will need to assess spares like a supermarket rather than a planned maintenance output

## **CONCLUSION**

The CMA's defined and considered within this paper differ significantly from the works of Nowlan and Heap and the SAE Standard JA1011. This of itself is not a problem, however, because of the fundamental flaw in the underlying assumption made by CMA's, that more maintenance will always provide greater reliability, the decisions reached by a CMA may be counterproductive.

The inaccuracy of the input data and variables driving the mathematical calculations exacerbates the situation.

These matters are likely to result in the following:

- Lower reliability and higher cost plant performance than could be achieved by using true RCM,
- Lack of confidence in the CMA recommendations leading to serious problems with implementation,
- Badly focussed planning and scheduling,
- Badly focussed spares assessing, and
- Difficulty in creating a "best practice" long-term reliability improvement program.

The decision logic defined by Nowlan and Heap and SAE JA1101 is simple and based largely on empirical data that is less sensitive to error. The methods of setting maintenance intervals can be easily proven and understood. They set a solid framework for planning and scheduling, spares assessing and reliability engineering.

It is unlikely that any CMA will provide a better solution to maintenance analysis than that provided by a decision diagram based on the works of Nowlan and Heap.

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### EXPLANATION OF DETERMINING THE INTERVAL OF INSPECTION

By Dana Netherton – Chairman of the SAE Standards Committee for RCM. (email to the plant maintenance resource group dated 29 May 2001)

So ... how do we figure out the interval at which an on-condition task is technically feasible ... the interval at which the task will \*work\*? The answer to that question is hidden in the description I gave for "on-condition tasks", a few minutes ago: "routine or cyclic tasks intended to detect the warning sign of an impending failure". How often do you need to look for a warning sign? Well, how far ahead do we look for warning signs in other settings?

By definition, all of us on this mailing list use e-mail. So suppose that your boss is going to send you an e-mail notifying you of a very important meeting in an hour -- but you don't know \*when\* he will send you the e-mail. How often should you check your e-mail? Once a week? Once a day? (No, more often) Or should you \*really\* check it at least once an hour? (And possibly every 30 minutes?) (Yes)

Suppose that he averages one such meeting each month -- his "mean time between meetings" (MTBM) is one month. Would that change your decision to check your e-mail at least once an hour? If his MTBM were one week, would that change your decision? (Of course not)

Now, let me connect this parable to maintenance. That hour's interval is the "warning time" your on-condition task gives you. Your e-mail check is your "on-condition task". The "mean time between meetings" is your asset's MTBF.

If you want to get an adequate warning from your on-condition tasks, you should not let the intervals between them go longer than the warning time they give you. No matter how often the failures might happen. Of course, we still need to ask whether doing our task \*at that interval\* is worth doing. And, as a reminder, that is where we might use MTBF -- to decide whether or not the task is worth doing \*at that interval\*, \*if\* the consequences are economic.

But MTBF did not enter into our initial effort to identify an interval that is technically feasible. And also BTW different technical approaches to on-condition tasks may offer different warning times. For example, vibration readings on a bearing will probably offer a longer warning time than listening for audible noise. If listening for audible noise (fairly often) is not worth doing, then taking vibration readings (less often) \*might\* be worth doing -- unless taking the vibration readings is significantly more expensive than listening for audible noise. Which is often the case, isn't it? The more sophisticated inspection methods usually offer longer warning times, but are also usually more expensive. \*shrug\* Sometimes you just have to crunch the numbers and find out.

## MATHEMATICS TO DETERMINE THE RELATIONSHIP BETWEEN PLANNED MAINTENANCE COSTS AND UNPLANNED MAINTENANCE COSTS

### Hypothesis:

That there is no point in conducting condition monitoring inspections at an interval greater than the PF interval.

### Definitions:

Condition Monitoring	The act of determining the condition of an asset and leaving it in service on the condition that will last at least until the next inspection.
PF Interval	The interval between the occurrence of a potential failure and its decay into a functional failure.

### Variables

Assign the following variables

T	The total time of evaluation (in years).
"t"	The interval of successive condition monitoring inspection (in days).
P	The PF Interval (in days).
MTBF	Mean Time Between Failure (in years)
$C_i$	The expected average cost of a single condition monitoring inspection over time T.
$C_f$	Expected average cost of a single failure
$C_{fu}$	The expected average cost of a single unplanned failure over time T. This cost includes any costs associated with lost production and repair.
$C_{fp}$	The expected average cost of a single planned failure over time T. This cost includes any costs associated with lost production and repair.
Pr(f)	The probability of failure
Pr( $C_{fu}$ )	The probability of an unplanned failure given and inspection interval t and a PF interval of P.
Pr( $C_{fp}$ )	The probability of a planned failure given and inspection interval t and a PF interval of P.
$C_T$	The total expected average cost over Time T
S	The probability of detecting the failure in one inspection given that it can be detected.

**Assumptions:**

T is much greater than t.

Because the inspection is fail safe, if  $t < P$  then  $Pr(C_{fu})=0$ . This is to say that there will be no unplanned failures regardless of the cost if the inspection is 100% effective or fail-safe and the interval of inspection is inside the PF Interval. More inspection will only add cost so the cheapest option in theory would be to inspect at  $t = P$ .

**Analysis**

First consider the case where the assessment method is fail safe. This means that it is 100% effective or reliable

To assess the hypothesis, let  $t > P$

Consider T to be 1 year and use the probability of failure to account for failures that may not occur in that particular year.

$C_T = (365/t)C_i + Pr(C_{fu})C_{fu} + Pr(C_{fp})C_{fp}$	Eq 1
$Pr(C_{fu}) = Pr(f) \times [(t-P)/t]$ for $t > P$ otherwise $Pr(C_{fu}) = 0$	Eq 2
$Pr(C_{fp}) = Pr(f) \times P/t]$ for $t > P$ otherwise $Pr(C_{fp}) = 1$	Eq 3
From Eq 1	
$C_T = (365/t)C_i + Pr(f) \times C_{fu} \times [(t-P)/t] + Pr(f) \times C_{fp} \times P/t$	Eq 4
$C_T = (1/t)[365C_i + Pr(f) \times C_{fu} \times (t-P) + Pr(f) \times C_{fp} \times P]$	Eq 5
$= (1/t)[365C_i - Pr(f) \times C_{fu} \times P + Pr(f) \times C_{fp} \times P] + Pr(f) \times C_{fu}$	Eq 6
$C_T = A/t + B$ where A and B are constants	Eq 7
<p><math>C_T</math> is therefore an hyperbolic function of t. This means that there is no point at which the costs change from increasing to decreasing (no point of inflection). This relationship is graphed at Figure A1.</p> <p>As t moves to infinity, the costs will converge to B. Whether the costs will ever be greater or less than B depends on whether A is positive or negative when <math>t = P</math>.</p> <p>The movement of total costs depends on whether A is positive or negative. If A is positive then as time progresses the total cost <math>C_T</math> will descend to B. Conversely, if A is negative, then the total cost will ascend to B.</p> <p>It is worth noting that the equation for A being negative is valid for <math>t &gt; p</math> as explained in Equation 2.</p>	
<p><b>The conclusion is that for <math>t &gt; P</math>, there is no lowest cost interval of inspection other than <math>t=P</math> or <math>t = infinity</math>.</b></p>	

Consider the implications of an inspection not being 100 % effective.

If inspections were not 100% effective then increasing the numbers of inspections could be worthwhile. This would add to the cost of the inspection but not change the costs of planned or unplanned failures. If it has already been established that inspection is not cost effective if it were 100% reliable, then a less reliable task would always be a more expensive option. This leads to the conclusion that condition monitoring is never optimal at an interval outside of the PF interval regardless of the effectiveness of the task. It may however, be optimised within the PF interval though this is a second order effect where task effectiveness is low and the cost of failure is high.

